
$\underline{\text { SECTION - A }}$
Answer ALL the following questions

1. Explain 'Confirmatory Analysis'.
2. Define Correlation matrix of a random vector.
3. Explain p-p plots.
4. Give an example to show that marginal normality does not assure joint normality.
5. Define partial correlation coefficient and give an expression for it in the case of multivariate normal distribution.
6. For a $4 \times 4$ correlation matrix, if the eigen values are $1.95,1.45,0.45$ and 0.15 , find the percentage of variance explained by the top two principal components.
7. Briefly explain the objective of Factor Analysis.
8. State any two distance measures for pairing of items / objects.
9. Mention any one way of scaling the coefficients of Fisher's discriminant function.
10. Explain the context for Multivariate Analysis of Variance.

## SECTION - B

Answer any FIVE questions
11. If $\mathbf{X}=\left(\mathrm{X}_{1}, \mathrm{X}_{2}\right)$ have the pmf given by the following table:

| $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | 0 |
| ---: | :---: | :---: |
| -1 | 0.08 | 0.22 |
| 0 | 0.18 | 0.12 |
| 1 | 0.30 | 0.10 |

Find the Mean Vector, Var-Cov Matrix, Correlation Matrix of $\mathbf{X}$.
12. Describe scatter plot enhancement with regression lines.
13. If $\mathrm{X}=\left[\begin{array}{l}X^{(1)} \\ X^{(2)}\end{array}\right] \sim \mathrm{N}_{\mathrm{p}}(\mu, \Sigma)$ and $\mu$ and $\Sigma$ are accordingly partitioned as $\left[\begin{array}{l}\mu^{(1)} \\ \mu^{(2)}\end{array}\right]$ and $\left[\begin{array}{ll}\Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22}\end{array}\right]$, derive the conditional distribution of $X^{(1)}$ given $X^{(2)}$.
14. Derive the moment generating function of $\mathrm{N}_{\mathrm{p}}(\mu, \Sigma)$.
15. Present the motivation for Principal components. Give its formal definition and derive the same for a random vector, stating without proof the lemma on maximization of a quadratic form on the unit sphere.
16. Explain the algorithm for hierarchical agglomerative methods of clustering. Explain any two linkages in this context.
17. Present the motivation and derive the Fisher's Discriminant Function for discriminating between two populations.
18. Derive the Hotelling's $\mathrm{T}^{2}$ statistic for testing hypothesis concerning the mean vector of a multivariate normal population.

## SECTION - C

## Answer any TWO questions

 $(\underline{2 \times 20=40}$ marks)19. (a) Give the motivation for multivariate normal distribution from univariate normal distribution and develop its p.d.f.
(b) Derive the MLEs of $\mathrm{N}_{\mathrm{p}}(\mu, \Sigma) \quad(10+10)$
20. (a) Discuss the Principal Factor Method of Estimation for the Parameters of the Factor model.
(b) Present the Regression Method of estimating the factor scores. $(10+10)$
21. (a) Explaining the notations, enlist any four similarity measures for pairs of items when variables are binary and state their rationale.
(b)Apply the single linkage process for clustering six objects whose distance matrix is given below (Dendrogram not required):

|  | 1 | 2 | 3 | 4 | 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 [0 |  |  |  |  |  |  |
| 220 |  |  |  |  |  |  |
| $\begin{equation*} 3 \tag{8+12} \end{equation*}$ | 2 | 1 | 0 |  |  |  |
| 4 | 7 | 5 | 6 | 0 |  |  |
| $5$ | 6 | 4 | 5 | 5 | 0 |  |
|  |  |  |  |  |  |  |

22. (a) Define Multiple Correlation Coefficient and derive an expression for it in the case of multivariate normal distribution.
(b) Obtain the optimal classification rules for two populations when the objective is to minimize cost of misclassification.
